* R F - 3 4 9 0 / 3 0 0 *

RF-3490

M. Sc. (Part - I) Examination

April / May – 2010

Mathematics : Paper - 404

(Complex Analysis)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. SC. - 1**

Name of the Subject :
☛ **MATHEMATICS - 404**

☛ Subject Code No. : **3 4 9 0** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Notations used are standard.

(3) Figure at the right end of the first line of each question indicate full marks.

Q.1	**(a)**	If f(z) = u(x,y)+iv(x,y) is differentiable at $\alpha = x_0 + iy_0$ then show that u and v satisfy the Cauchy-Riemann equation.	[6]
	(b)	Prove that if γ is a closed path, then winding number is an integer.	[5]
	(c)	Define composite function. Also state and prove chain rule.	[3]
		OR	
Q.1	**(a)**	Prove that if γ is a path then the function of α defined by $\alpha \to \int_\gamma \frac{dz}{z-\alpha}$ for α not on the path is a continuous function of α.	[6]
	(b)	Find the radius of convergence of the series (i) $\sum \frac{z^n}{n^n}$. (ii) $\sum \frac{z^n}{n!}$	[5]
	(c)	State and prove the Integral Estimate theorem.	[3]
Q.2	**(a)**	State and prove Cauchy-Goursat theorem.	[8]
	(b)	Let $\{f_n\}$ be a sequence of analytic function on an open set U converging uniformly on every compact subset K of U to a function f. Show that the sequence $\{f'_n\}$ of derivatives of f converges uniformly on K and $\lim_{n\to\infty} f'_n = f'$.	[6]
		OR	

Q.2	**(a)**	Let γ be path in an open set U and g be a continuous function on γ. If z is not on γ and $f(z)=\int_{\gamma}\frac{g(\xi)}{\xi-\alpha}d\xi$. Show that f is analytic on the complement of γ in U and hence prove that $f^{n}(z)=n!\int_{\gamma}\frac{g(\xi)}{(\xi-\alpha)^{n+1}}d\xi$.	[8]
	(b)	Evaluate $\int_{\gamma}xdz$, where (i) γ is the line segment from 0 to a + ib. (ii) γ is the circle $\lvert z\rvert = R$.	[6]
Q.3	**(a)**	State and prove Cauchy Residue formula.	[6]
	(b)	Let $f(z)=z^2-2z+3$ and C be a rectangle containing the points $1+i\sqrt{2}$, $1-i\sqrt{2}$ in the interior. Find $\int_{C}\frac{1}{f(z)}dz$.	[5]
	(c)	Evaluate $\int_{\lvert z\rvert=1}\frac{e^{iaz}}{z^4}dz$	[3]
		OR	
Q.3	**(a)**	Prove that $\int_{0}^{\infty}\frac{\cos(ax)-\cos(bx)}{x^2}dx=\frac{\pi}{2}(b-a)$	[6]
	(b)	Prove that $\int_{0}^{\infty}\frac{x\sin(mx)}{x^2+a^2}dx=\frac{\pi e^{-ma}}{2}(m>0)$	[5]
	(c)	Suppose $f(z_0)=0$ but $f'(z_0)\neq 0$ then $\frac{1}{f}$ has a pole of order 1 at z_0 and residue of $\frac{1}{f}$ is $\frac{1}{f'(z_0)}$.	[3]
Q.4	**(a)**	State and prove Poissons formula.	[6]
	(b)	State Green's theorem. Also for a function f analytic on R and on its boundary C, show that Green's theorem implies Cauchy's theorem.	[5]
	(c)	Let f and g be analytic functions on a connected open set U. Suppose f and g have the same real parts then f = g+ik.	[3]
		OR	
Q.4	**(a)**	Let f be an entire function of strict order $\leq\rho$. Let $V_{\rho}(R)$ be the number of zeros of f in the disc of radius R. Show that $V_{\rho}(R)\ll R^{\rho}$.	[6]

	(b)	Let u be a harmonic function on an open set U. Let $z_0 \in U$ and let r > 0 be a number such that the closed disc of radius r with centre z_0 contained in U. Then $f(z_0)=\frac{1}{2\pi}\int_0^{2\pi} f\left(z_0+re^{i\theta}\right)d\theta$.	[5]
	(c)	Let f be holomorphic on a closed annulus $0 < r_1 < \|z\| < r_2$. Let $M(r) = \|\|f\|\|_r = max\|f(x)\|$ on $\|z\| = r$ then $logM(r) \leq (1-s)logM(r_1) + slogM(r_2)$.	[3]
Q.5	**(a)**	Prove that $\cot(\pi z) = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{1}{z^2-n^2}$.	[6]
	(b)	Define Harmonic and Sub-Harmonic functions. Also for a harmonic function u, show that u^2 is sub-harmonic.	[5]
	(c)	Show that if u_1, u_2 are sub-harmonic and c_1, c_2 are positive numbers then $c_1u_1 + c_2u_2$ is sub-harmonic.	[3]
		OR	
Q.5	**(a)**	Show that $\sin(\pi z) = \pi z\prod_{n=1}^{\infty}\left(1-\frac{z^2}{n^2}\right)$.	[6]
	(b)	The product $\prod_{n=1}^{\infty} E_n(z, z_n) = \prod_{n=1}^{\infty}\left(1-\frac{z}{z_n}\right)e^{P_n\left(\frac{z}{z_n}\right)}$ converges uniformly and absolutely on every disc $\|z\| \leq R$ defines an entire function with zero's at the points of sequence $\{z_n\}$ and no other zero's.	[5]
	(c)	Let f be an entire function without zero's then there exists an entire function h(z) such that $f(z) = e^{h(z)}$	[3]